

## Combined Similarity-numerical Solutions of MHD Boundary Layer Slip Flow of Non-Newtonian Power-law Nanofluids over a Radiating Moving Plate

(Gabungan Penyelesaian Keserupaan Persamaan-berangka Aliran Slip bagi Lapisan Sempadan MHD dengan Nano Bendalir Hukum Kuasa yang Bukan-Newtonian atas Permukaan Beradiasi yang Bergerak)

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### ABSTRACT

*A combined similarity-numerical solution of the magnetohydrodynamic boundary layer slip flow of an electrically conducting non-Newtonian power-law nanofluid along a heated radiating moving vertical plate is explored. Our nanofluid model incorporates the influences of the thermophoresis and the Brownian motion. The basic transport equations are made dimensionless first and then suitable similarity transformations are applied to reduce them into a set of nonlinear ordinary differential equations with the associated boundary conditions. The reduced equations are then solved numerically. Graphical results for the non-dimensional flow velocity, the temperature and the nanoparticles volume fraction profiles as well as for the friction factor, the local Nusselt and the Sherwood numbers are exhibited and examined for various values of the controlling parameters to display the interesting aspects of the solutions. It was found that the friction factor increases with the increase of the magnetic field ( $M$ ), whilst it is decreased with the linear momentum slip parameter ( $a$ ). The linear momentum slip parameter ( $a$ ) reduces the heat transfer rates and the nanoparticles volume fraction rates. Our results are compatible with the existing results for a special case.*

*Keywords: Magnetic field; momentum slip boundary condition; non-Newtonian power-law nanofluids; radiation*

### ABSTRAK

*Gabungan penyelesaian keserupaan-berangka aliran tergelincir bagi lapisan sempadan magnetohidrodinamik nano bendalir hukum kuasa yang bukan-Newtonian yang boleh mengalirkan elektrik atas permukaan bergerak serta beradiasi diterokai. Model nanobendalir kami menggabungkan pengaruh termoforesis dan gerakan Brownian. Persamaan pengangkutan asas dijadikan tidak berdimensi dahulu dan transformasi persamaan yang sesuai digunakan untuk mengurangkan ke satu set persamaan pembezaan biasa tak linear dengan syarat sempadan yang berkaitan. Persamaan yang dikurangkan kemudian diselesaikan secara berangka. Keputusan grafik untuk halaju tidak berdimensi, suhu, profil pecahan isi padu zarah-zarah nano, nombor Nusselt serta Sherwood dipamerkan dan dikaji bagi pelbagai parameter kawalan untuk memaparkan aspek-aspek yang menarik daripada penyelesaian. Kajian mendapati bahawa regangan dinding ricih meningkatkan peningkatan medan magnet  $M$ , manakala menurun dengan momentum lurus slip parameter  $a$ . Momentum lurus slip parameter ( $a$ ) mengurangkan kadar pemindahan haba dan pecahan isi padu zarah nano. Keputusan kami adalah selaras dengan keputusan yang sedia ada bagi kes khas.*

*Kata kunci: Keadaan sempadan momentum slip; medan magnet; nanobendalir yang bukan-Newtonian; radiasi*

### INTRODUCTION

Non-Newtonian nanofluid is important in many industrial and technological applications such as biological solutions, melts of polymers, paint, tars and glues (Ellahi et al. 2012). Because of this, researches on non-Newtonian fluids have recently become very important. Transport phenomena associated with magnetohydrodynamics arise in physics, geophysics, astrophysics and many branches of chemical engineering which includes crystal magnetic damping control, hydromagnetic chromatography; conducting flow in trickle-bed reactors and enhanced magnetic filtration control (Prasad et al. 2010). Many experimental and numerical studies associated with magnetohydrodynamics transport in porous media regimes have been reported in the literature (Lioubashevski et al. 2004). Sakiadis (1961) has

investigated the boundary layer flow due to a continuous solid surface. Crane (1970) has studied steady boundary layer flow past a stretching sheet. Following Crane (1970) many investigators have extended his problem in various aspects (Boutros et al. 2006; Mahapatra et al. 2007).

The traditional no slip boundary conditions at the solid fluid interface is not valid for the fluid flow in a micro electro mechanical system and must be replaced by slip boundary condition. The slip flow model expresses a connection between the tangential component of the velocity at the surface and the velocity gradient normal to the surface (Hak 2002). Ellahi (2009) has examined the effect of slip condition at the plate on the flow of an Oldroyd-B constant fluid. The uniqueness and existence of the steady flow of an incompressible fluid of third-grade

fluid due to slip and no slip boundary condition in bounded domain has been proven by Roux (2009). Very recently, Noghrehabadi et al. (2012) have analysed the slip effects on the boundary layer flow and heat transfer of nanofluids which is due to a stretching surface.

Many industrial and environmental processes encounter the radiative heat flow. It is important in controlling the heat transfer in the polymer processing industry. The qualities of the final products depend on the heat controlling factors and hence the knowledge of radiative heat transfer in the system can assist in this regard (Mahmoud 2011). Raptis et al. (2004) have investigated the problem of the radiative flow in the presence of the magnetic field. Besides that, Cortell (2008) has studied the Blasius flow with the thermal radiation effects. In another paper, Cortell (2011) has studied radiation effects in the presence of a free stream velocity for a power-law fluid past an infinite porous plate. Very recently, Hady et al. (2012) have investigated the flow and heat transfer characteristic of a viscous nanofluid over a nonlinearly stretching sheet in the presence of the thermal radiation.

The motions of the particles that influence by thermal gradient is known as thermophoresis. The transport due to temperature is due to the fact that in the presence of a temperature gradient in the atmosphere. A particle is pushed towards the lower temperature because of the asymmetry of molecular impacts (Piazza & Parola 2008). Blackening of chimneys and industrial furnaces are examples of thermophoresis. Thermophoresis is a mass transfer mechanism in the modified chemical vapor deposition process used in the fabrication of optical fiber and is important in radioactive particle deposition in nuclear reactors. Putra et al. (2003) have investigated the natural convection of nanofluids inside a horizontal cylinder heated from one end and cooled from the other experimentally. Elhajjar et al. (2010) have investigated the Rayleigh-Benard natural convection heat transfer for three types of nanofluids. Very recently, Yu and Xie (2012) have reported that nanofluids can be utilized in, for example, areas such as heat transfer, electronic applications, industrial cooling applications, reducing pollution, nuclear systems cooling, mass transfer enhancement, energy storage, reduction of friction, magnetic sealing, biomedical applications and nanodrug delivery. Magnetic nanofluids have important industrial and biomedical applications (Guo et al. 2010). The steady boundary layer flow of a nanofluid along an exponential stretching surface was investigated by Nadeem and Lee (2012). The Cheng and Minkowycz (1977) problem for natural convective flow along a vertical plate in a porous medium filled with a nanofluid was investigated by Nield and Kuznetsov (2011). Very recently, Uddin et al. (2012a) have studied free convective boundary layer flow of a nanofluid along a convectively heated vertical plate with the linear hydrodynamic slip boundary condition.

From the literature survey, it seems that no research has been carried out on MHD boundary layer slip flow of non-Newtonian nanofluid past a moving radiating plate with

the momentum slip boundary condition. Therefore, our present study aimed to explore the numerical solutions of the transformed nonlinear decoupled ordinary differential equations for different values of the controlling parameters. The model introduced by Buongiorno (2006) has been used in the present study. The effect of the controlling parameters on the dimensionless velocity, temperature, nanoparticles volume fraction, the rate of heat transfer and the rate of a nanoparticles volume fraction will be shown graphically and discussed.

## MATHEMATICAL MODEL

Consider a steady two-dimensional laminar viscous incompressible flow of an electrically conducting non-Newtonian power-law nanofluid past a radiating moving vertical solid flat plate. The flow configuration and coordinates system is shown in Figure 1 (i stands for momentum boundary layer and ii and iii stand for thermal and solute boundary layers, respectively). It is assumed that the plate is located in an infinitely large body of fluid of constant properties. We assumed that the uniform wall temperature  $T_w$  and nanoparticles volume fraction  $C_w$  are higher than that of their full stream values  $T_\infty, C_\infty$ . It is further assumed that a magnetic field of uniform of strength  $B_0$  act to the perpendicular to the plate. The describing boundary layer equations in dimensional form are (Buongiorno 2006):

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \quad (1)$$

$$\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{K}{\rho} \left( \frac{\partial \bar{u}}{\partial \bar{y}} \right)^n - \frac{\sigma B_0^2}{\rho} \bar{u}, \quad (2)$$

$$\begin{aligned} \bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = & \left( \alpha + \frac{16\sigma_r T_\infty}{3(\rho c)_p K_1} \right) \frac{\partial^2 T}{\partial \bar{y}^2} \\ & + \tau D_B \frac{\partial T}{\partial \bar{y}} \frac{\partial C}{\partial \bar{y}} + \tau \left( \frac{D_T}{T_\infty} \right) \left( \frac{\partial T}{\partial \bar{y}} \right)^2, \end{aligned} \quad (3)$$

$$\bar{u} \frac{\partial C}{\partial \bar{x}} + \bar{v} \frac{\partial C}{\partial \bar{y}} = D_B \frac{\partial^2 C}{\partial \bar{y}^2} + \left( \frac{D_T}{T_\infty} \right) \frac{\partial^2 T}{\partial \bar{y}^2}. \quad (4)$$

The boundary conditions are taken as (Mahmoud 2011):

$$\begin{aligned} \bar{u} &= \bar{u}_w + \bar{u}_{slip} = U_r \left( \frac{\bar{x}}{L} \right) + \beta_1 \left( -\frac{\partial \bar{u}}{\partial \bar{y}} \right)^{n-1} \cdot \frac{\partial \bar{u}}{\partial \bar{y}}, \\ \bar{v} &= 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad \bar{y} = 0, \\ \bar{u} &= 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as} \quad \bar{y} \rightarrow \infty. \end{aligned} \quad (5)$$

Here  $(\bar{u}, \bar{v})$  are the velocity components along and perpendicular to the plate,  $\tau = \frac{(\rho c)_p}{(\rho c)_f}$  is the ratio of nanoparticles heat capacity and the base fluid heat capacity,  $\alpha = \frac{k}{(\rho c)_f}$  is the thermal diffusivity of the fluid,  $U_r$  is the

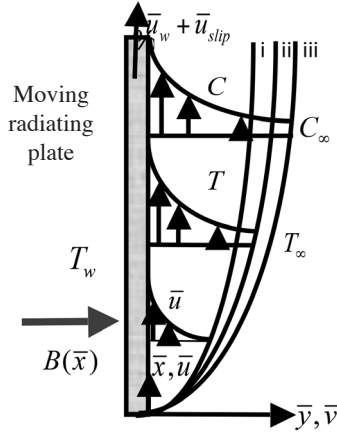


FIGURE 1. Flow configuration and coordinate system

reference velocity,  $K$  is the consistency coefficient,  $K_1$  is the Rosseland mean absorption coefficient,  $\sigma_1$  is the Stefan-Boltzman constant,  $D_B$  is the Brownian diffusion coefficient,  $D_T$  is the thermophoretic diffusion coefficient,  $\beta_1$  is the momentum slip factor with appropriate dimension,  $k$  is the thermal conductivity,  $\rho$  is the fluid density,  $\sigma$  is the electric conductivity and  $B_0$  is the magnetic field.

It is interesting to note that for  $n = 1$  (Newtonian fluid), with the dynamic coefficient of viscosity  $\mu = K$ , our model is consistent with Makinde and Aziz (2011) and Noghrehabadi et al. (2012). If  $n < 1$  the model represents shear thinning fluids (pseudoplastic) and if  $n > 1$  the model represent shear thickening fluids (dilatant).

#### NONDIMENSIONALIZATION OF THE TRANSPORT EQUATIONS

Introducing the following dimensionless variables,

$$\begin{aligned} x &= \frac{\bar{x}}{L}, & y &= \frac{\bar{y}}{L} \text{Re}^{\frac{1}{n+1}}, & u &= \frac{\bar{u}}{U_r}, \\ v &= \frac{\bar{v}}{U_r} \text{Re}^{\frac{1}{n+1}}, & \theta &= \frac{T - T_\infty}{T_w - T_\infty}, & \phi &= \frac{C - C_\infty}{C_w - C_\infty}, \end{aligned} \quad (6)$$

where  $\text{Re} = \frac{\rho U_r^{2-n} L^n}{K}$  is the Reynolds number and hence introduce the stream function  $\psi$  which is defined by:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad (7)$$

into (2) - (5) lead to:

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = n \left( -\frac{\partial^2 \psi}{\partial y^2} \right)^{n-1} \frac{\partial^3 \psi}{\partial y^3} - M \frac{\partial \psi}{\partial y}, \quad (8)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \frac{1+R}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} + \text{Nb} \frac{\partial \theta}{\partial y} \frac{\partial \phi}{\partial y} + \text{Nt} \left( \frac{\partial \theta}{\partial y} \right)^2, \quad (9)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y} = \frac{1}{\text{Le}} \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{\text{Le}} \frac{\text{Nt}}{\text{Nb}} \frac{\partial^2 \theta}{\partial y^2}, \quad (10)$$

subject to the boundary conditions:

$$\frac{\partial \psi}{\partial y} = x + \beta_1 \frac{U_r^{n-1} \text{Re}^{\frac{n}{n+1}}}{L^n} \left( -\frac{\partial^2 \psi}{\partial y^2} \right),$$

$$\frac{\partial \psi}{\partial x} = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad y = 0,$$

$$\frac{\partial \psi}{\partial y} = 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty. \quad (11)$$

The parameters are  $M$  (magnetic field),  $R$  (radiation),  $\text{Pr}$  (Prandtl number),  $\text{Nb}$  (Brownian motion),  $\text{Nt}$  (thermophoresis) and  $\text{Le}$  (Lewis number). The parameters are defined as:

$$M = \frac{\sigma B_0^2 L}{\rho U_r}, \quad \text{Le} = \frac{1}{D_B} \left( \frac{U_r^{3n-3} L^{1-n} K}{\rho} \right)^{\frac{2}{n+1}},$$

$$\text{Nt} = \tau (T_w - T_\infty) \left( \frac{D_T}{T_\infty} \right) \left( \frac{U_r^{3-3n} L^{n-1} \rho}{K} \right)^{\frac{2}{n+1}},$$

$$\text{Nb} = \tau D_B (C_w - C_\infty) \left( \frac{U_r^{3-3n} L^{n-1} \rho}{K} \right)^{\frac{2}{n+1}},$$

$$\text{Pr} = \frac{U_r}{\alpha L} \left( \frac{\rho U_r^{2-n}}{KL} \right)^{\frac{2}{n+1}}, \quad R = \frac{16\sigma_1 T_\infty^3}{3\rho c_p K \alpha}. \quad (12)$$

We introduce the following similarity transformations developed by group transformation (Uddin et al. 2012b),

$$\eta = yx^{\frac{1-n}{1+n}}, \quad \psi = x^{\frac{2n}{1+n}} f(\eta), \quad \theta = \theta(\eta), \quad \phi = \phi(\eta). \quad (13)$$

Using (13) into (8)-(11), we have:

$$f'''' + \frac{2}{n+1} (-f'')^{(1-n)} f f'' - \frac{1}{n} (-f'')^{(1-n)} f'^{(2)} - \frac{1}{n} M (-f'')^{(1-n)} f' = 0, \quad (14)$$

$$\left( \frac{1+R}{\text{Pr}_m} \right) \theta'' + \frac{2n}{n+1} f \theta' + \text{Nb}_m \phi' \theta' + \text{Nt}_m \theta'^{(2)} = 0, \quad (15)$$

$$\phi'' + \frac{\text{Nt}_m}{\text{Nb}_m} \theta'' + \frac{2n}{n+1} \text{Le}_m f \phi' = 0. \quad (16)$$

The corresponding boundary conditions are:

$$\begin{aligned} f(0) &= 0, & f'(0) &= 1 + a (-f'')^{n-1}(0) f''(0), \\ \theta(0) &= \phi(0) = 1, & f'(\infty) &= \theta(\infty) = \phi(\infty) = 0. \end{aligned} \quad (17)$$

Here  $\text{Le}_m = \text{Le} x^{\frac{2n-2}{n+1}}$  (modified Lewis number),  $\text{Nb}_m = \text{Nb} x^{\frac{2n+2}{n+1}}$  (modified Brownian motion parameter),  $\text{Nt}_m = \text{Nt} x^{\frac{-2n+2}{n+1}}$  (modified thermophoresis parameter),  $\text{Pr}_m = \text{Pr} x^{\frac{-2n+2}{n+1}}$  (modified Prandtl number) and  $a = \beta_1 \frac{x^{\frac{1-n}{1+n}} U_r^{n-1} \text{Re}^{\frac{n}{n+1}}}{L^n}$

(velocity slip parameter). For a Newtonian fluid ( $n = 1$ ), in the absence of radiation ( $R = 0$ ) and magnetic field ( $M = 0$ ), our problem reduces and so as Noghrehabadi et al. (2012).

The physical quantities we interested in are the local skin friction factor ( $C_{f\bar{x}}$ ), the local Nusselt number ( $Nu_{\bar{x}}$ ) and the local Sherwood number ( $Sh_{\bar{x}}$ ) which are defined as:

$$\begin{aligned}
 C_{f\bar{x}} &= \frac{K}{\rho \bar{u}_w^2} \left( \frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0}, \\
 Nu_{\bar{x}} &= \frac{-\bar{x}}{(T_w - T_\infty)} \left( \frac{\partial T}{\partial \bar{y}} \right)_{\bar{y}=0}, \\
 Sh_{\bar{x}} &= \frac{-\bar{x}}{(C_w - C_\infty)} \left( \frac{\partial C}{\partial \bar{y}} \right)_{\bar{y}=0}.
 \end{aligned} \tag{18}$$

Using (6) and (13), the dimensionless parameters can be written in terms of the output of the similarity solutions as:

$$\begin{aligned}
 \text{Re}_{\bar{x}}^{\frac{1}{n+1}} C_{f\bar{x}} &= [f''(0)]^n : \text{reduced friction factor,} \\
 \text{Re}_{\bar{x}}^{\frac{1}{n+1}} Nu_{\bar{x}} &= -\theta'(0) : \text{reduced Nusselt number,} \\
 \text{Re}_{\bar{x}}^{\frac{1}{n+1}} Sh_{\bar{x}} &= -\phi'(0) : \text{reduced Sherwood number.}
 \end{aligned} \tag{19}$$

where  $\text{Re}_{\bar{x}} = \left( \frac{\rho \bar{u}_w^{2-n} \bar{x}^n}{K} \right)$  is the local Reynolds number.

### RESULTS AND DISCUSSION

Equations (14) to (16) along with the boundary conditions in (17) are solved numerically by the Runge-Kutta-Fehlberg fourth fifth order numerical method. The computations are carried out using Maple 14. The obtained numerical results are presented graphically to highlight salient features of the flow, the heat transfer and the nanoparticle volume fraction transfer characteristics. Table 1 shows the comparison of our results with Xue and Liao (2009) and Andersson and Bech (1992). A good agreement is found. This supports the validity of our numerical results for other cases. In all the figures we remove subscripts  $m$  from the parameters  $Le_m, Nb_m, Nt_m, Pr_m$  for simplicity. Figure 2(a) shows the effect of power law index  $n$  on the dimensionless velocity profiles in the absence of  $R$  and  $a$ . It is found that the

dimensionless velocity decreases with the increasing of  $n$ . For  $n < 1$  (0.5, 0.7, 0.9) the non-Newtonian nanofluid exhibits shear-thinning properties (pseudoplasticity) and possesses lower viscosity than Newtonian fluids ( $n = 1$ ). Increasing values of  $n$  from 0.5 through 0.7 to 0.9 lead to an increase in the viscosity. Increasing viscosity leads to decrease in the fluid velocity. The effect of the increasing values of  $n$  is to reduce the boundary layer thickness. Figure 2(b) shows the velocity decreases with the rising of  $M$ . This is due to the magnetic field opposing the transport phenomena, since the variation of  $M$  causes the variation of the Lorentz force. Lorentz force is a drag-like force that produces more resistance to transport phenomena and that causes reduction in the fluid velocity (Prasad et al 2012). Figure 2(c) shows the effect of the linear momentum slip boundary condition on the velocity in the flow field. Fluid velocity is maximum for  $a = 0$  (no slip). The increment of the linear momentum slip parameter causes the velocity to decrease. Physically this is because the increasing slipping factor may be looked at miscommunication between the source of motion (the plate) and fluid domain. This effect has been shown to be significant in the manufacturing of complex liquids as described by Rosenbaum and Hatzikiriakos (1997). Figure 3(a) interprets the effect of  $n$  whilst Figure 3(b) displays influence of  $M$ . A significant decrease in temperature accompanies a rise in  $n$  from 0.5 through 0.7 to 0.9 (Figure 3(a)). With an increasing viscosity of the nanofluid, thermal diffusion is depressed in the regime which cools the boundary layer and decreases the thermal boundary layer thickness. The temperature is found to increase as  $M$  increases. This is due to the increase in the wall temperature gradient which in turn increases the surface heat transfer. Physically, fluids with smaller Prandtl number  $Pr$  have a larger thermal diffusivity. Hence, the effect of  $M$  is more efficient for fluids with a smaller Prandtl number  $Pr$  and the fluids are more sensitive to the magnetic force compare to fluids which have a larger Prandtl number  $Pr$ . The rise of  $a$  as in Figure 3(c) shows that there is an increase in temperature profile due to reduction of fluid velocity. It is noticed from Figure 3(d) that the temperature of the fluid increases with the increment value of  $R$ . As expected, an increase of the radiation has the tendency to increase effects of conduction as well as to increase the temperature at each point away from the surface. Hence, higher values of  $R$  imply a higher

TABLE 1. Comparison of  $-f''(0)$  - for various  $n$  with Xue and Liao (2009) and Andersson and Bech (1992) when  $a = M = R = 0$

$n$	Current results	Xue and Liao (2009)	Andersson and Bech (1992)
0.4	1.27296	-	1.273
0.6	1.09643	-	1.096
0.8	1.02853	1.02853	1.029
1.0	1.00000	1.00000	1.000
1.2	0.98738	-	0.987

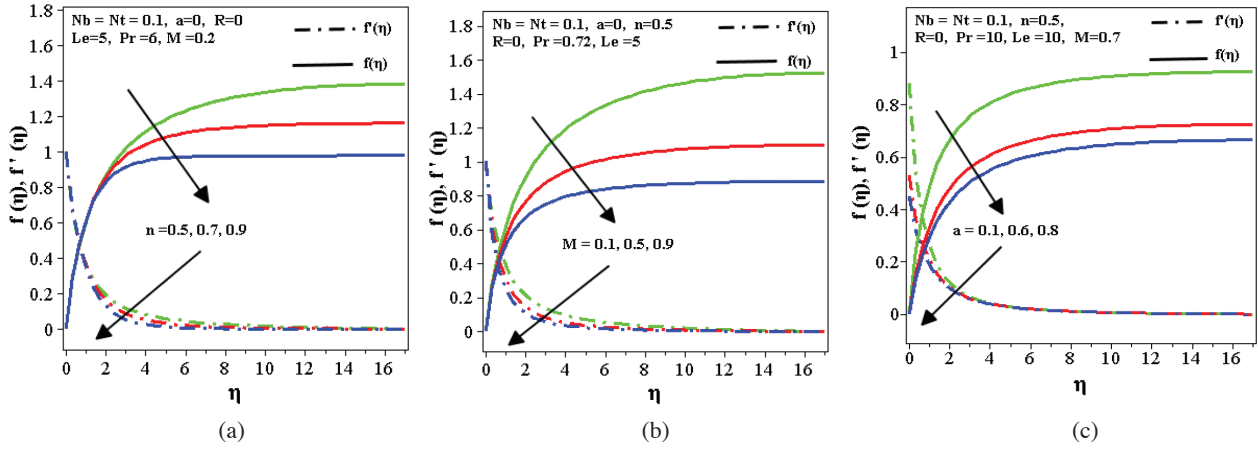


FIGURE 2. Variations of the dimensionless transverse and axial velocity profiles with  $n$ ,  $M$  and  $a$

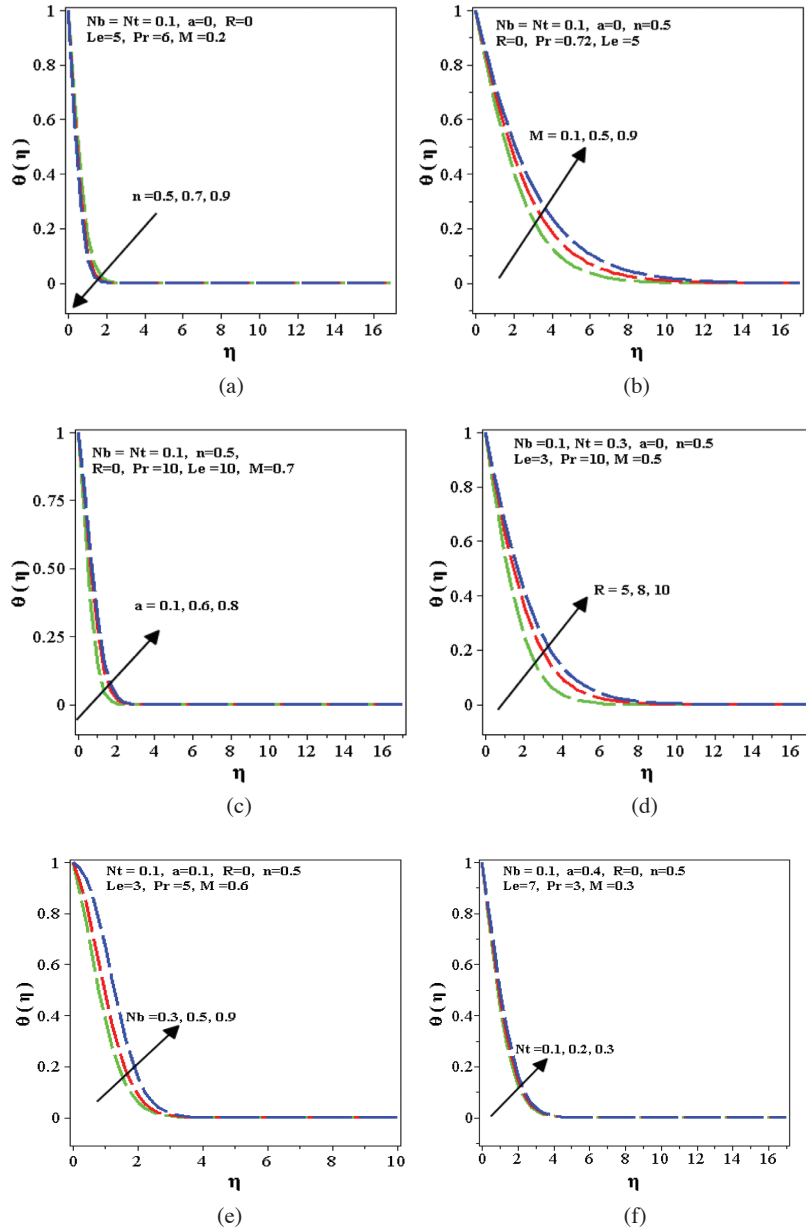
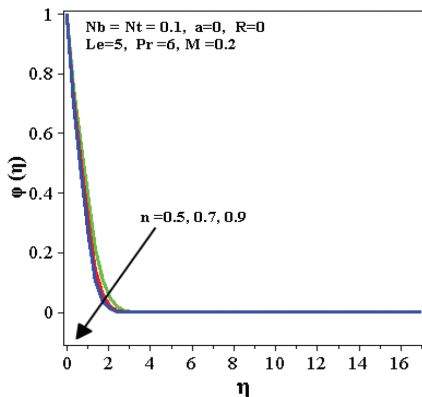


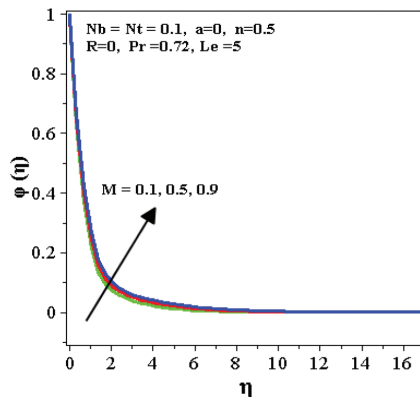
FIGURE 3. Variations of the dimensionless temperature profiles with  $n$ ,  $M$ ,  $a$ ,  $R$ ,  $Nb$  and  $Nt$

surface heat flux. Note that  $Le$  effect on temperature is negligible. This may be due to the combine effect of the other parameters. The dimensionless temperature profiles increase with an increase in the  $Nb$  and  $Nt$  (Figure 3(e), 3(f)). It can be observed from Figure 4(a) that, the increment of  $n$  reduces the nanoparticles volume fraction. Increasing  $n$  from 0.5 to 0.9, strongly decreases the nanoparticles volume fraction i.e. decreases diffusion of nano-particles and this manifests as a thinning in the nanoparticle volume fraction (concentration) boundary layer thickness. From Figure 4(b), the rise of  $M$  causes the increment in nanoparticles volume fraction, since there is a reduction of the fluid velocity. The rate of nanoparticles volume fraction transfer decreases due to the increase of the nanoparticles volume fraction. Figure 4(c) shows that increasing  $a$  leads to increase in the dimensionless nanoparticles volume fraction as nanoparticles volume fraction distribution is driven by temperature distribution. The dimensionless concentration is decreased due to the increasing of  $Le$  can be observed in Figure 4(d). From the definition of Lewis number, a higher value of Lewis number cause a lower Brownian motion coefficient  $D_B$  having a kinematic viscosity,  $\frac{K}{\rho}$ . Due to that, higher Lewis number reduces the nanoparticles volume fraction

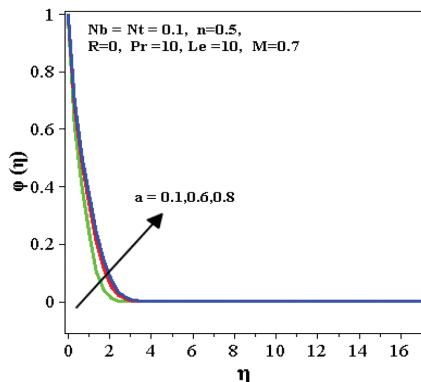
and its boundary layer thickness. It is noticed from Figure 4(e), a cross flow occur at  $h = 2868$  (approx.). The behaviour of nanoparticles volume fraction after cross flow is opposite of before cross flow. This cross flow may be due to the combined effects of all parameters. Note that the nanoparticles volume fraction profiles decreases with an increase in  $Nb$  whilst it increases with an increase in  $Nt$ . From Figure 5 we note that the skin friction coefficient increases with the increasing of  $M$ . It is found that friction factor decreases with momentum slip parameter  $a$ . The boundary condition connecting the velocity field to the wall shear stress in (17) i.e.  $|f''(0)|^n = \frac{f'(0)-1}{a}$  implies that a positive increase in  $a$  accompanied with a shear stress *decrease* will effectively reduce the flow velocity at the wall and this will manifest in a flow deceleration with increasing momentum slip effect, throughout the boundary layer regime. Conversely the presence of progressively greater momentum slip at the wall, by opposing momentum development in the body of the fluid, (i.e. inducing flow deceleration) will enhance thermal diffusion and this will lead to an increase in temperatures. This is observed in Figure 5. Figure 6 shows the effect of the Prandtl number  $Pr$  and the linear



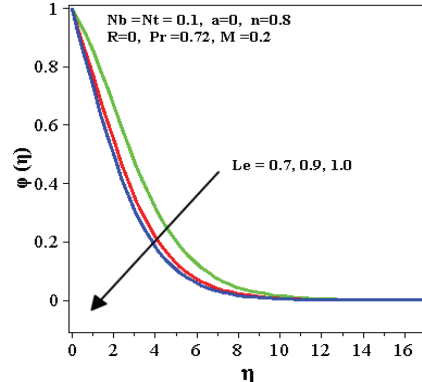
(a)



(b)



(c)



(d)

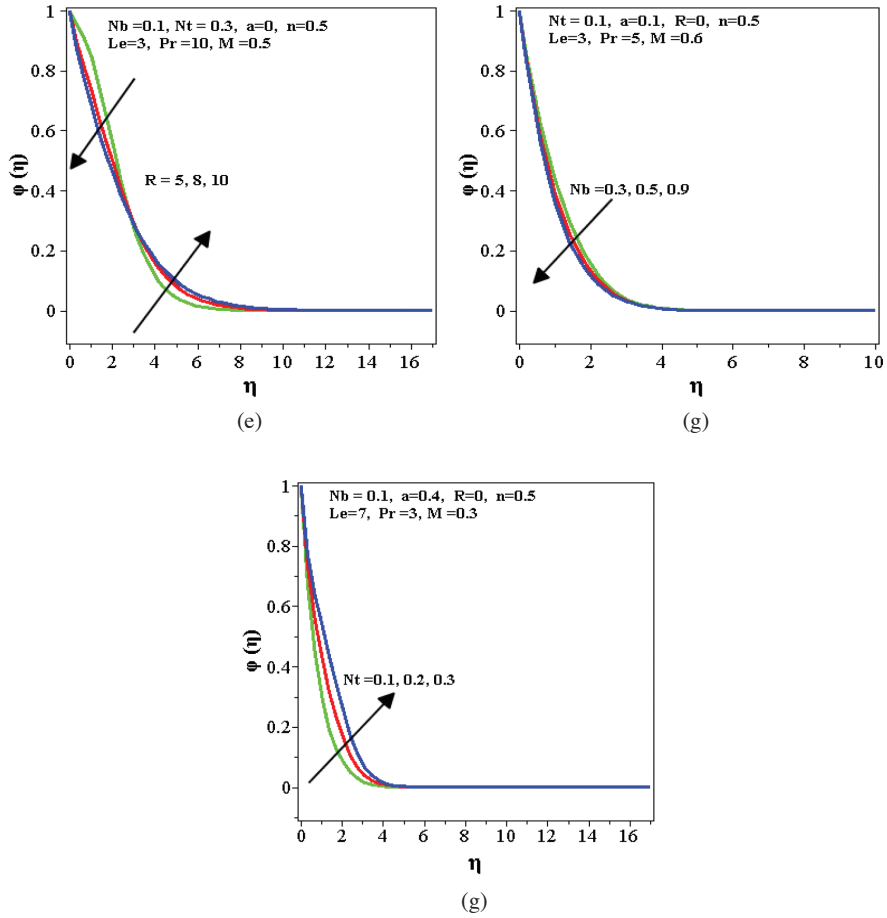


FIGURE 4. Variations of the dimensionless nanoparticles volume fraction profiles with  $n, M, a, R, Le, Nb$  and  $Nt$

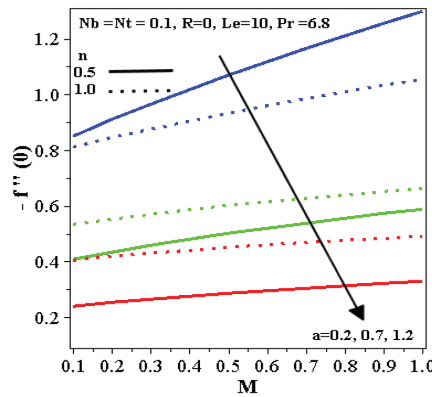


FIGURE 5. Variation of the friction factor with  $M$  and  $a$  for different power law index

momentum slip parameter  $a$  on the dimensionless heat transfer rate for different values of  $n$ . It can be seen that dimensionless heat transfer rate decreases with increasing of  $a$ . This is due to the fact that increasing  $a$  induces less amount of fluid due to the moving plate. This reduction in the fluid induced motion makes the transfer of convection

modes stronger than the conduction modes. It is further noticed that heat transfer rates increases with  $Pr$  and  $n$ , as expected. Figure 7 shows that like heat transfer rates, the nanoparticles volume fraction rates also decreases with increase of  $a$  whilst it increases with  $Le$  and  $n$ .

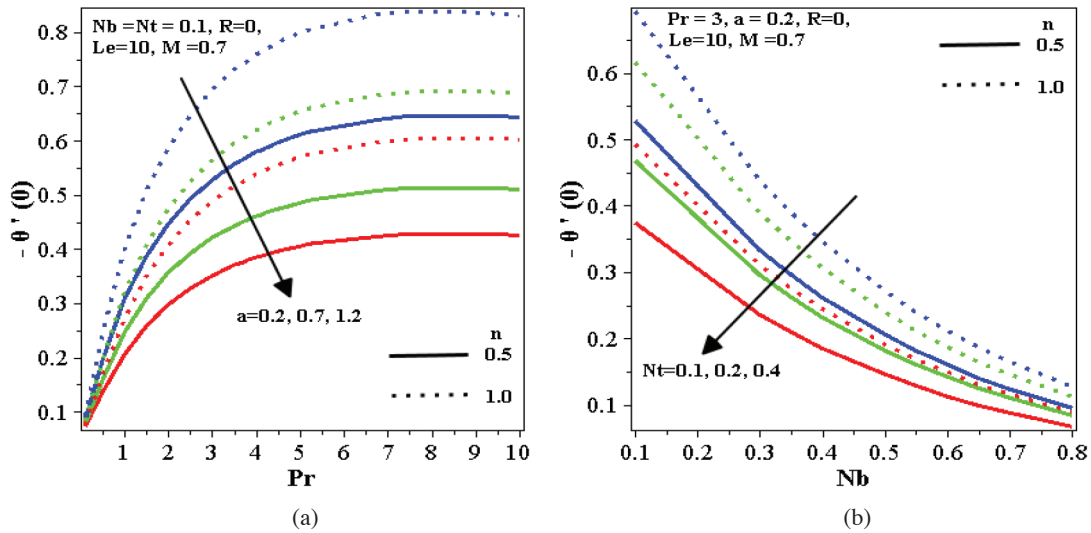


FIGURE 6. Variation of the heat transfer rates with  $Pr$ ,  $a$ ,  $Nb$  and  $Nt$  for different power law index

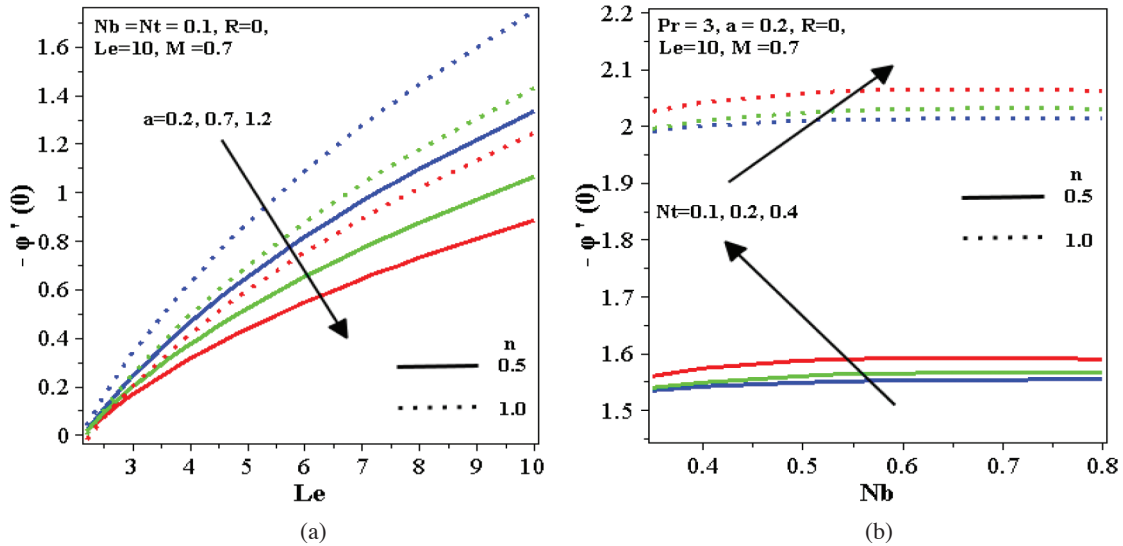


FIGURE 7. Variation of the nanoparticles volume fraction rates with  $Le$ ,  $a$ ,  $Nb$  and  $Nt$  for different power law index

CONCLUSION

A combined similarity-numerical solution of MHD boundary layer slip flow of non-Newtonian power-law nanofluids along a moving radiating vertical flat plate is explored. It was concluded that the dimensionless velocity, the temperature and the nanoparticles volume fraction decrease with  $n$ . The dimensionless velocity decreases while the temperature and the nanoparticles volume fraction increase with  $M$ . The dimensionless velocity decreases while the temperature and the nanoparticles volume fraction increase with  $a$ . The dimensionless nanoparticles volume fraction decreases with  $Le$ . The dimensionless temperature increases with  $R$ . The friction factor increases with the increase of  $M$ , whilst it is decreased with  $a$ . The heat transfer rate increases with  $n$  and  $Pr$  decreases with  $a$ ,  $Nb$ ,

$Nt$ . The nanoparticles volume fraction rate increases with  $Le$  and  $n$  and decreases with  $a$ .

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